We have all heard the expression that what goes up must come down. There are exceptions. If a rocket has thrust that is greater than its weight, it will accelerate away from the Earth. Another exception is a rocket or any object coasting away from the Earth, with enough speed, so that it never returns. Escape velocity is the minimum velocity an object in free-fall can have and still not be pulled back to Earth. We can determine the escape velocity of an object leaving any planet or star by using the principle of conservation of energy. Let’s use conservation of energy to calculate the escape velocity of a projectile leaving the Earth.

Objects exposed to a gravitational field will possess gravitational potential energy.

\[ G.P.E \equiv mgh \]

where \( m \) is the mass of an object, \( g \) is the acceleration due to gravity and \( h \) the the height above the center of mass of the Earth. The value of \( g \) depends on the Earth’s mass and the distance from its center as seen in the following equation:

\[ g = \frac{\gamma M_E}{R^2} \]

\( \gamma \) is known as the universal gravitational constant and its value is \( 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \)

The mass of the Earth, \( M_E = 5.98 \times 10^{24} \text{ kilograms} \)

The radius of the Earth, \( R = 6.37 \times 10^6 \text{ meters} \)

The gravitational potential energy of an object with mass \( m \), at the earth’s surface is:

\[ G.P.E = mgh = m \frac{\gamma M_E}{R^2} R = \frac{m \gamma M_E}{R} \]
Escape velocity

\[ \text{The G.P.E.} = \frac{m\lambda M_E}{R} \]

The kinetic energy of an object is as follows:

\[ K.E. \equiv \frac{1}{2} m v^2 \], where m is the mass of the object and v is the object’s speed.

**From conservation of energy:**

The total energy of the object at the earth’s surface = the total energy at a distance r from the earth’s surface.

\[ GPE_{\text{surface}} + KE_{\text{surface}} = GPE_{\text{at } R+r} + KE_{\text{at } R+r} \]

\[ \frac{\gamma m M_E}{R} + \frac{1}{2} m v^2_{\text{surface}} = \frac{\gamma m M_E}{R + r} + \frac{1}{2} m v^2_{R+r} \]

Divide through by the mass of the object m.

\[ \frac{\gamma M_E}{R} + \frac{1}{2} v^2_{\text{surface}} = \frac{\gamma M_E}{R + r} + \frac{1}{2} v^2_{R+r} \]

At a large distance, R+r is a large number.

\[ \frac{\gamma M_E}{R + r} = 0 \]

Also note that if we are trying to determine the minimum escape velocity, we can set the velocity at R+r equal to 0.

\[ \frac{1}{2} v^2_{R+r} = 0 \]

Therefore,

\[ \frac{\gamma M_E}{R} + \frac{1}{2} v^2_{\text{surface}} = 0 + 0 \]
Escape velocity

\[ \frac{1}{2} v_{\text{surface}}^2 = \frac{\gamma M_E}{R} \]

Set the down direction as negative and up as positive.

\[ v_{\text{surface}} = \sqrt{\frac{2\gamma M_E}{R}} \]

Where \( v_{\text{surface}} \) is the escape velocity.

\[ v_{\text{surface}} = \sqrt{\frac{2\gamma M_E}{R}} = \sqrt{\frac{2 \cdot 6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{6.37 \times 10^6}} = 1.12 \times 10^4 \frac{m}{s} \]

Escape velocity at the earth’s surface is; \( 11.2 \frac{km}{sec} \)